An Efficient Integral Equation Solver for the Electromagnetic Modeling of Highly-Integrated Planar RF/Microwave Circuits

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***Abstract* — An efficient methodology for the accurate calculation of closed-form Green's function in multilayered planar media is combined with the adaptive integral method (AIM) to provide a fast, iterative, full-wave solver for the analysis of complex planar integrated circuits. The computational complexity per iteration and memory requirements for the AIM-based electromagnetic solver scale as O(NlogN) and O(N), respectively, where N is the number of unknowns in the discrete model. The accuracy and efficiency of the solver is demonstrated through its application to the modeling of an integrated, planar circuit component.**

I. INTRODUCTION

One of the primary obstacles in the effective prototyping and design of system-in-a-package or system-on-chip designs is the large circuit density and the associated strong electromagnetic coupling between both passive and non-passive components and their associated interconnect network. More specifically, circuit components cannot be designed independently anymore; rather a global electromagnetic modeling is required that takes into account all electromagnetic interactions with adjacent components in order to effect subsystem or system design optimization and/or performance verification. Furthermore, the mixed-signal attributes of these highly integrated designs further impound the electromagnetic complexity of the problem since electromagnetic interactions and their impact on component, subsystem and system performance must be calculated over a very broad frequency bandwidth. Finally, it is mentioned that, contrary to narrowband microwave modeling of isolated components, it is the fineness of the feature size rather than sufficient wavelength resolution that becomes the dominating factor in the choice of the grid size for the development of the discrete model.

From a computational complexity point of view the aforementioned attributes of the electromagnetic analysis of high-density, mixed-signal circuits, results in discrete models involving very large numbers of unknowns even when integral equation methods are used for the electromagnetic analysis. In addition to the time-consuming and often times computationally prohibitive direct inversion of the resulting linear system of equations, the storage of the matrix itself becomes an issue of concern. To address these issues, several methodologies have been proposed over the past several years toward the fast solution of the matrix equation resulting from the discretization of the electromagnetic integral equation [1], [2]. One of the issues that has not been resolved entirely in the context of these fast solvers is the way the electromagnetic Green's function is calculated in the case where the circuitry is embedded in a multilayered, planar dielectric substrate. As already mentioned, memory storage is at a premium when dense circuits are modeled. Hence, rather than storing the method of moments matrix, its calculation on the fly during the iterative solution process is desired. This, in turn, requires a fast, yet accurate calculation of the layered media Green's function.

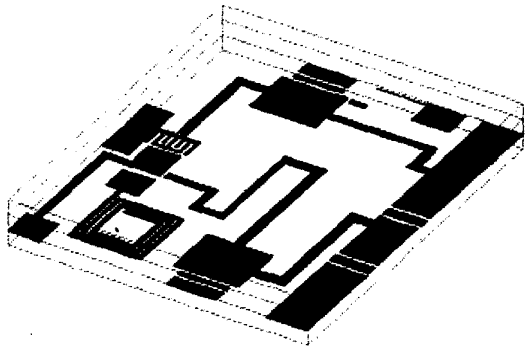

Fig. 1. Generic geometry of a planar microstrip structure emerged in layered media.

This issue is addressed in this paper. More specifically, a highly accurate closed-form expression is derived for the layered media Green's function, based on the extension of the methodology presented in [3]. This closed-form Green's function is used in conjunction with the adaptive integral method for the electromagnetic analysis of integrated circuits in layered media. The accuracy and efficiency of the resulting solver are examined through its application to the modeling of a generic planar integrated circuit.

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II. INTEGRAL EQUATION SCHEME ACCELERATED BY AIM

Assuming that the circuit structure is planar, we reduce the boundary value problem to the set of integral equations,

$$\int_S G^{xx}(\vec{r},\vec{r}')\cdot J_x(\vec{r}')ds' + \int_S G^{xy}(\vec{r},\vec{r}')\cdot J_y(\vec{r}')ds' = E_x^{inc}(\vec{r}) \quad (1a)$$

$$\int_S G^{yx}(\vec{r},\vec{r}')\cdot J_x(\vec{r}')ds' + \int_S G^{yy}(\vec{r},\vec{r}')\cdot J_y(\vec{r}')ds' = E_y^{inc}(\vec{r}) \quad (1b)$$

Description of the layered media Greens function G is given in the next Section. Excitation of the circuit is modeled using either impressed voltage or impressed current models presented in [4]. Using method of moments (MoM), equations (1) can be solved numerically. As basis functions for the unknown current expansion we choose regular roof-top functions

$$J_u(x,y) = \sum_{n=1}^{N_u} I_n^{(u)} b_n^{(u)}(x,y) \ , \qquad (2)$$

where u assumes the values x and y. In order to avoid hypersingular integration we move the derivatives acting on the singular kernel to the testing functions $t_n^{(u)}(x,y)$ by choosing testing functions to be smooth and twice differentiable with respect to both x and y coordinates. As a result, the discrete approximation of (1) is obtained in the form of the following set of linear equations

$$\begin{bmatrix} \mathbf{Z}^{xx} & \mathbf{Z}^{xy} \\ \mathbf{Z}^{yx} & \mathbf{Z}^{yy} \end{bmatrix} \begin{bmatrix} \mathbf{I}^x \\ \mathbf{I}^y \end{bmatrix} = \begin{bmatrix} \mathbf{V}^x \\ \mathbf{V}^y \end{bmatrix}, \qquad (3)$$

where Z_{mn}^{uv} is the scattered field produced by the n th basis function of v orientation and tested with the m th testing function of u orientation. Vectors $\mathbf{I}^x$ and $\mathbf{I}^y$ are the unknown coefficients in the expansion, and the vectors $\mathbf{V}^x$ and $\mathbf{V}^y$ describe the weighted forms of the excitation. For electrically large or dense circuits, iterative methods (e.g. conjugate gradient) are needed to solve (3), where the computational complexity of such methods is dominated by the $O(N^2)$ matrix-vector product in each iteration.

AIM addresses this complexity by splitting $\mathbf{Z}\cdot\mathbf{I}$, the product of the MoM matrix and vector of current expansion coefficients, into "near-zone" and "far-zone" interactions. This splitting may be cast in the form

$$\mathbf{Z}\cdot\mathbf{I} = \mathbf{Z}_{MoM}^{near}\cdot\mathbf{I} + \mathbf{Z}^{far}\cdot\mathbf{I} = \mathbf{Z}_{MoM}^{near}\cdot\mathbf{I} + \left(\mathbf{Z}_{AIM}\cdot\mathbf{I} - \mathbf{Z}_{AIM}^{near}\cdot\mathbf{I}\right). \quad (4)$$

In the above equation, $\mathbf{Z}_{MoM}^{near}\cdot\mathbf{I}$ contains the interactions between closely spaced elements separated by distances less than some threshold distance and calculated using standard MoM. Clearly, the matrix $\mathbf{Z}_{MoM}^{near}$ is very sparse and the computational complexity of $\mathbf{Z}_{MoM}^{near}\cdot\mathbf{I}$ is $O(N)$.

The "far-zone" interactions are treated in a very different manner. First, a rectangular grid is laid over the entire area occupied by the circuit. Each of the basis and testing functions, $b_n^{(u)}(x,y)$ and $t_n^{(u)}(x,y)$, in the current expansions are replaced by M^2 equivalent "delta" sources,

$$b_n^{(u)}(\vec{r}) \simeq \hat{b}_n^{(u)}(\vec{r}) = \sum_{m_1=1}^{M-1}\sum_{m_2=1}^{M-1} B_{m_1,m_2,n}^{(u)}\,\delta(x-x_{m_1,n}^{(u)})\delta(y-y_{m_2,n}^{(u)}), \quad (5a)$$

$$t_n^{(u)}(\vec{r}) \simeq \hat{t}_n^{(u)}(\vec{r}) = \sum_{m_1=1}^{M-1}\sum_{m_2=1}^{M-1} T_{m_1,m_2,n}^{(u)}\,\delta(x-x_{m_1,n}^{(u)})\delta(y-y_{m_2,n}^{(u)}), \quad (5b)$$

where $\{x_{m_1,n}^{(u)}, y_{m_2,n}^{(u)}\}$ are the locations of the delta sources associated with the nth basis or testing functions [1]. Substitution of (5) into the integrals that define the elements of the MoM matrix yields

$$Z_{mn}^{uv} = \sum_{k_1=0}^{K_1-1}\sum_{k_2=0}^{K_2-1}\sum_{k_1'=0}^{K_1-1}\sum_{k_2'=0}^{K_2-1} T_{k_1,k_2,m}^{(u)}\, G^{uv}\left((k_1-k_1')\Delta x,(k_2-k_2')\Delta y\right) B_{k_1',k_2',n}^{(v)},$$

where $m = 1,\ldots N_u; n = 1,\ldots N_v$. $\qquad (6)$

In the above equation, K_1, K_2 are the total number of the AIM grid points along x and y, respectively. Since only M^2 delta sources are used in (5), the matrices $B^{(u)}$ and $T^{(u)}$ each have only $M^2\cdot N_u$, $u = x$ or y, non-zero elements. Thus, the matrix-vector product in the iterative solution of the MoM equation may be computed as

$$\mathbf{Z}_{AIM}\cdot\mathbf{I} = \begin{bmatrix} T^{(x)}\mathbf{G}^{xx}B^{(x)T}\mathbf{I}^x + T^{(x)}\mathbf{G}^{xy}B^{(y)T}\mathbf{I}^y \\ T^{(y)}\mathbf{G}^{yx}B^{(x)T}\mathbf{I}^x + T^{(y)}\mathbf{G}^{yy}B^{(y)T}\mathbf{I}^y \end{bmatrix}. \qquad (7)$$

Because the Green's function for interactions between the delta sources on the rectangular grid has a convolutional form, the complexity of (7) is $O(N\log N)$ and the product can be computed using FFT. The product $\mathbf{Z}_{AIM}\cdot\mathbf{I}$ is computed at each iteration (see (4)).

While the calculation of the "far-zone" interactions using the AIM process is accurate, the calculated "near-zone" interactions are not. Thus, the operation $\mathbf{Z}_{AIM}\cdot\mathbf{I} - \mathbf{Z}_{AIM}^{near}\cdot\mathbf{I}$ is required to correct the calculation of the "near-zone" interactions by replacing the AIM calculations with those obtained using the "exact" MoM representation of the expansion function interactions. These operations are of $O(N)$ complexity.

AIM implementation also relaxes memory requirements since only the MoM matrix elements that describe the "near-zone" are stored. Avoiding the $O(N^2)$ storage of the MoM matrix results in memory requirements of $O(N)$. Furthermore, since Green's function matrices are Toeplitz, the memory requirements also scale as $O(N)$.

III. GREEN'S FUNCTION OF PLANAR LAYERED MEDIA

In [3], a closed-form formula for the Green's function in planar stratified media was given as a finite series of Hankel functions. This representation was found by using finite-difference scheme to solve for the vector potentials in the spectral domain, which leads to analytic evaluation of Sommerfeld integrals. Briefly presented here is a similar approach based on the expansion of the vector potentials in terms of Chebyshev polynomials. The method is valid for arbitrary orientation of exciting electric or magnetic dipole.

As an example, let us consider an $\hat{x}$ directed electric dipole located at $x' = y' = 0$ and embedded in N dielectric layers stacked in $\hat{z}$. To describe the dipole field it is necessary to consider two components of vector potential. Here, as in [3], the magnetic vector potential is taken to be $\mathbf{A} = A_x\hat{x} + A_z\hat{z}$, where $\mathbf{H} = \nabla \times \mathbf{A}$. After applying Fourier - Bessel transform to the scalar Helmholtz equations for the vector potentials, and defining an auxiliary potential as $\Lambda_z = {\partial A_z}/{\partial x}$, we arrive at the following equations to solve in the spectral domain

$$\frac{d^2\tilde{A}_x}{dx^2} + (k^2 - \lambda^2)\tilde{A}_x = \frac{-I_x\ell}{2\pi}\delta(z - z'), \quad \frac{d^2\tilde{\Lambda}_z}{dx^2} + (k^2 - \lambda^2)\tilde{\Lambda}_z = 0. \tag{8}$$

Within each layer, the solution of both $\tilde{A}_x$ and $\tilde{\Lambda}_z$ is expanded in terms of $M+1$ Chebyshev polynomials of order zero to M. Substitution of this expansion into (8) and applying a point collocation technique using the Gauss-Labatto collocation points, the solution in each layer can be written in the form

$$\left[\mathbf{D}^2 + k^2\mathbf{I}\right]\tilde{\mathbf{a}} - \left[\lambda^2\mathbf{I}\right]\tilde{\mathbf{a}} = -\mathbf{f}. \tag{9}$$

Here $\mathbf{I}$ is the $(M+1)\times(M+1)$ identity matrix, $\tilde{\mathbf{a}}$ is a vector of $2(M+1)$ unknowns corresponding to the potentials at the collocation points, $\mathbf{f}$ is a vector of all zeros except for the collocation point corresponding to z' if it exists in this layer, and $\mathbf{D}^2$ is the square of the Chebyshev collocation derivative matrix for the Gauss-Labatto points given in [5]. It should be noted that the dipole must be located at an interface between two layers in order for the solution to have a discontinuous derivative at z'. If the dipole does not already reside at a layer interface, an "artificial" interface must be added at this location. The N equations in the form of (9) are then combined into a single $2N(M-1)\times 2N(M+1)$ matrix equation in block form, where the equations at collocation points corresponding to the layer interfaces have been removed. To couple together the N matrix blocks, the unknowns corresponding to potentials located at the layer interfaces will be removed by use of the boundary conditions [3]. The boundary conditions at the layer interfaces are

$$\tilde{A}_x^+ = \tilde{A}_x^-, \ \tilde{\Lambda}_z^+ = \tilde{\Lambda}_z^-, \ \frac{d\tilde{A}_x^+}{dz} - \frac{d\tilde{A}_x^-}{dz} = \frac{-I_x\ell}{2\pi}\delta_{zz'},$$
$$\varepsilon^+\left(\tilde{A}_x^- + \frac{d\tilde{\Lambda}_z^-}{dz}\right) = \varepsilon^-\left(\tilde{A}_x^+ + \frac{d\tilde{\Lambda}_z^+}{dz}\right) \tag{10}$$

The + and – represent the solutions at the interface from above and below respectively, and $\delta_{zz'} = 1$ at the interface corresponding to the dipole location. These boundary conditions can be found in [3] for interfaces where $\delta_{zz'} = 0$. To use these boundary conditions, we first note that the layer interfaces correspond to collocation points, and the z derivative of the potential at the i^{th} collocation points can be written as [5]

$$\frac{d\tilde{A}_{xi}}{dz} = \sum_{j=0}^{M}\frac{2\tilde{A}_{xj}D_{ij}}{d}. \tag{11}$$

In (11) d is the thickness of the layer, D_{ij} is the ij^{th} element of the derivative matrix $\mathbf{D}$, and $\tilde{A}_{xj}$ is potential at the j^{th} collocation point. Equation (11) is applied to (10) for both $\tilde{A}_x$ and $\tilde{\Lambda}_z$, and the resulting relationships are used to eliminate the unknown potentials at the interface locations. Once this is done, the system of $2N(M-1)$ unknowns can be written in the form

$$\left[\mathbf{A} + \lambda^2\mathbf{I}\right]\tilde{\mathbf{a}} = -\mathbf{f} \tag{12}$$

where $\mathbf{A}$ and $\tilde{\mathbf{a}}$ are now a full matrix and vector due to the application of (10). Eigenvalue decomposition on $\mathbf{A}$ gives pole-residual form for the spectrums of potentials. As a result, the inverse Fourier-Bessel transform provides a closed-form formula for the potential in spatial domain

$$\left.\begin{matrix} A_x(\rho,z,z') \\ \Lambda_z(\rho,z,z') \end{matrix}\right\} = \frac{j\pi}{M}\sum_{m=0}^{M}\sum_{n=0}^{M}\frac{T_m(z)T_m(z')}{c_m c_n}\sum_{k=1}^{2N(M-1)}P_{nk}R_k H_0^{(2)}(\sigma_k\rho). \tag{13}$$

Here $\sigma_k = \sqrt{|s_k|}\exp(-j/2\arg(s_k))$, T_m is the m th Chebyshev polynomial, the z_n's are the collocation points for the layer where the observation point z is located, $\mathbf{P}$ and $\mathbf{s}$ are the eigenvectors and eigenvalues of $\mathbf{A}$, and $\mathbf{R}$ is the matrix vector product $\mathbf{R} = \mathbf{P}^{-1}\cdot\mathbf{f}$. Coefficients c_m, c_n are equal to 2, if $m, n = 0$ or M and 1 otherwise.

Although three nested series appear in (13), the superior convergence of the Chebyshev polynomials, compared to a finite difference solution, allows for a smaller number of total unknowns that reduce the complexity of the eigenvalue decomposition. In addition, the use of Chebyshev polynomials allows for the derivative of Λ_z with respect to z to be more accurately taken as analytic derivatives of the Chebyshev polynomials.

VI. NUMERICAL RESULTS

To validate the proposed formulation of the Green's function in conjunction with the AIM algorithm, we considered various microwave components such as microstrip antennas, filters and multi-conductor transmission lines. On Fig.2 the input impedance of a low-pass filter considered in [6] versus frequency is presented. The solid line on the plot corresponds to the MoM solution based on the Green's function generated by discrete complex image method and the crosses correspond to the solution obtained by AIM using the Green's function described in this paper. It is important to notice that results in Fig. 2 were obtained using applied-voltage excitation model [4] without deembeding of the effects associated with truncation of the feeding lines. The lines were truncated at the length of 10mm on both sides of the filter and the second port was left open.

In Fig. 3 CPU time per iteration and memory storage required for MoM and AIM versus number of unknowns is shown. One can see that the memory used by AIM grows linearly with the number of unknowns N, while the memory required for a standard MoM solution grows as N^2. Such dramatic memory savings are possible because storage of the entire matrix $\mathbf{Z}$ is avoided in the AIM solution. It is also be noted that relatively few elements of $\mathbf{Z}$, only those associated with near interactions, are computed. Hence the AIM scheme fills the matrix in $O(N)$ CPU operations instead of $O(N^2)$ operations traditionally required by MoM.

In Fig. 3 we also demonstrate that AIM leads to $O(N\log(N))$ CPU time complexity per iteration for the matrix-vector product $\mathbf{Z}\cdot\mathbf{I}$, compared to $O(N^2)$ complexity associated with the method of moments. Clearly, for a low number of unknowns direct matrix-vector evaluation is faster, but time saving advantages for the iterative solver in the AIM implementation becomes obvious when the number of unknowns exceeds 10^3.

V. CONCLUSION

Fast algorithms become essential when the number of unknowns associated with a microwave circuit analysis problem is large. In this work we demonstrated how a new efficient Green's function evaluation scheme could be used in conjunction with AIM to provide fast and accurate tool for simulation of dense planar integrated circuits.

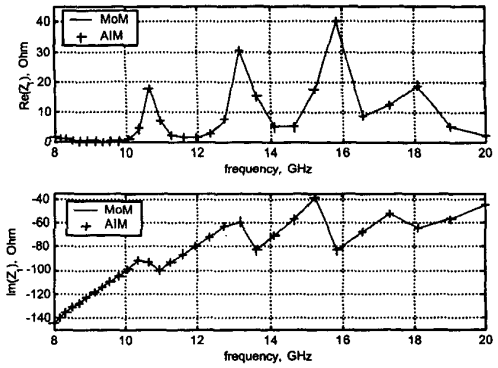

Fig. 2. Real and imaginary parts of low-pass filter input impedance.

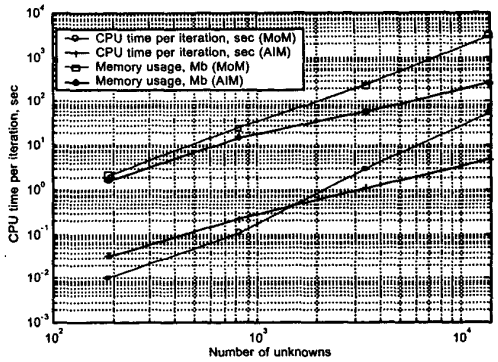

Fig. 3. CPU time per iteration and memory consumption versus number of unknowns

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